

Projectile Motion

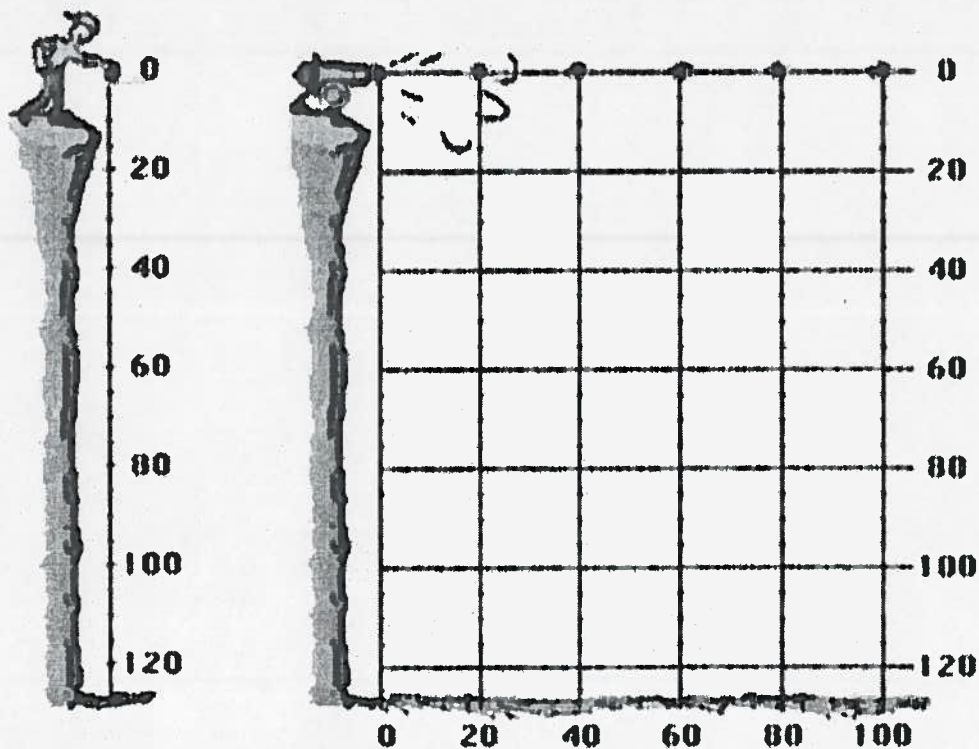
Read from Lesson 2 of the Vectors and Motion in Two-Dimensions chapter at The Physics Classroom



MOP Connection: Vectors and Projectiles: sublevels 7 - 10

1. A baseball is dropped off a cliff and it accelerates to the ground at a rate of -9.8 m/s^2 , down. Meanwhile a cannonball is launched horizontally from a cannon with a horizontal speed of 20 m/s .
2. A scale is shown along the sides of the graphic at the right. Use the scale to locate the position of the baseball and the cannonball. Trace a line to indicate the trajectory of the cannonball.

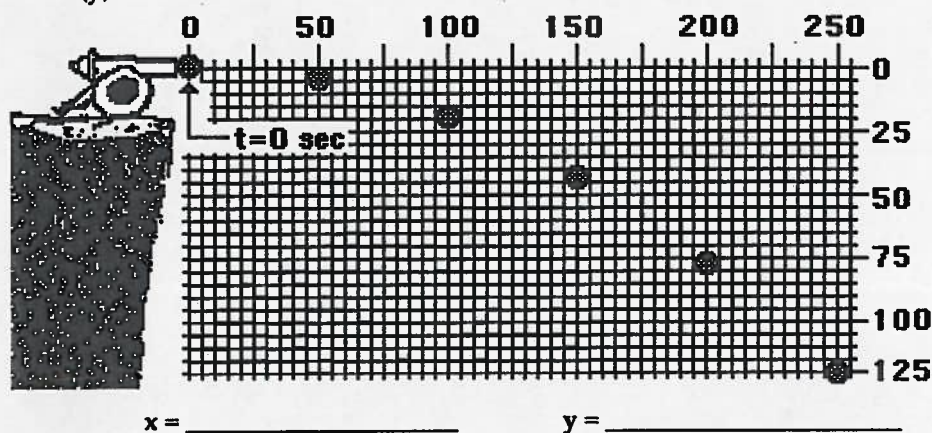
| Baseball | | Cannonball | | |
|----------|------|------------|------|------|
| t(s) | y(m) | t(s) | x(m) | y(m) |
| 0 | | | | |
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |



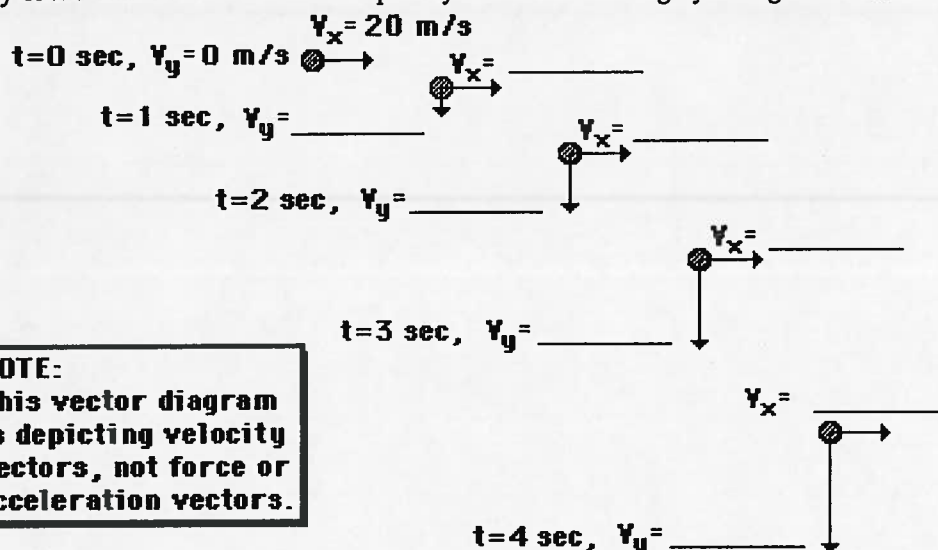
3. Which of these two balls strike the ground first? _____
4. Compare the two diagrams - the vertical free-fall motion on the left and the two-dimensional free fall motion on the right. Describe the effect on an object's horizontal motion upon the object's vertical motion.

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5. The diagram below shows the trajectory of a horizontally launched projectile. Positions of the projectile at 1-second intervals are shown. Demonstrate your understanding of the component displacement vector by determining the horizontal displacement (x) and the vertical displacement (y) after the fifth second.



6. A ball is launched horizontally from the top of a cliff with an initial velocity of 20 m/s. The trajectory of the ball is shown below. Express your understanding by filling in the blanks.



7. If the ball in the diagram above strikes the ground after four seconds, then (a) how high was the cliff and (b) how far from the base of the cliff will the ball land? **PSYW**
8. If the ball's initial speed in question #6 was 16 m/s, then how far from the cliff will the ball land?

9. Use the diagram below to construct a free-body diagram for a vertically launched projectile as it rises towards its peak, at its peak, and as it falls from its peak.

Before Peak:

At Peak:

After Peak:



10. Use the diagram below to construct a free-body diagram for a projectile launched at an angle as it rises towards its peak, at its peak, and as it falls from its peak.

Before Peak:

At Peak:

After Peak:



11. A projectile is launched with a speed of 31.1 m/s at an angle of 71.2 degrees above the horizontal. The horizontal and vertical components of the initial velocity are shown in the first row of the table. Fill in the table indicating the value of the horizontal and vertical components of velocity the projectile during the course of its motion.

| Time (s) | v_x (m/s) | v_y (m/s) | a_x (m/s ²) | a_y (m/s ²) |
|----------|-------------|-------------|---------------------------|---------------------------|
| 0 | 10.0 | 29.4 | | |
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |



REMEMBER

Key Concepts:

A projectile is an object which has the following characteristics.

- The only force acting on it is a gravitational force; it is a free-falling object.
- The acceleration is directed downwards and has a value of 9.8 m/s².
- Once projected, it continues its horizontal motion without any need of a force.
- As it rises, its vertical velocity (v_y) decreases; as it falls, its v_y increases.
- As it travels through the air, its horizontal velocity remains constant.

Vectors and Projectiles

The Equations:

Kinematic equations used for 1-dimensional motion can be used for projectile motion as well. The key to their use is to remember that perpendicular components of motion are independent of each other. As such, the equations for one dimension must be applied to either the horizontal motion of a projectile or the vertical motion of a projectile. When using the equations to analyze projectile motion, one assumes negligible air resistance and an acceleration of gravity of 9.8 m/s^2 , down(-). Thus, $a_x = 0 \text{ m/s}^2$ and $a_y = -9.8 \text{ m/s}^2$.

$$\text{1-Dim.} \quad v_f = v_o + a \cdot t \quad d = v_o \cdot t + \frac{1}{2} \cdot a \cdot t^2 \quad v_f^2 = v_o^2 + 2 \cdot a \cdot d \quad d = \frac{v_o + v_f}{2} \cdot t$$

$$\text{x-comp.} \quad v_{fx} = v_{ox} + a_x \cdot t \quad x = v_{ox} \cdot t + \frac{1}{2} \cdot a_x \cdot t^2 \quad v_{fx}^2 = v_{ox}^2 + 2 \cdot a_x \cdot x \quad x = \frac{v_{ox} + v_{fx}}{2} \cdot t$$

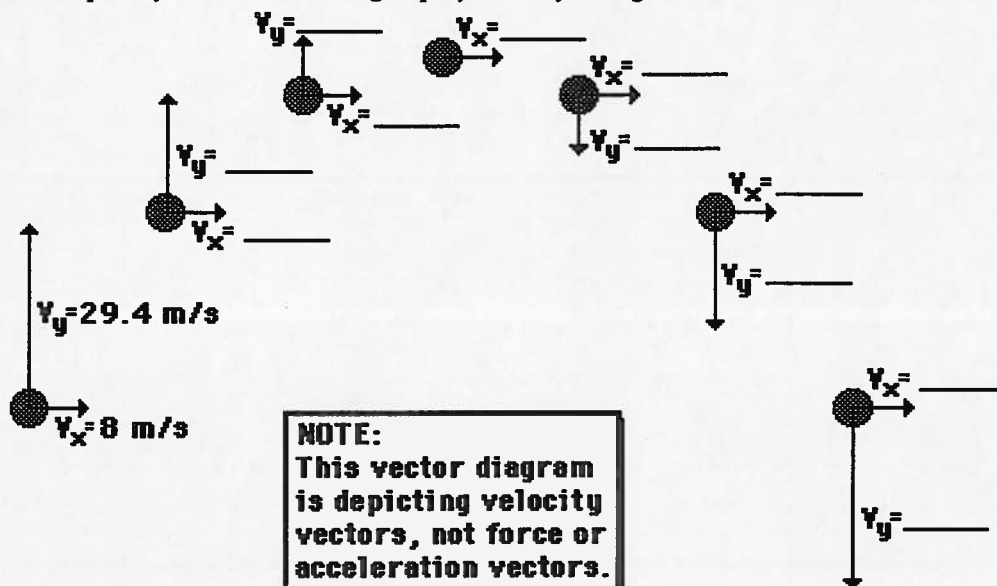
$$\text{y-comp.} \quad v_{fy} = v_{oy} + a_y \cdot t \quad y = v_{oy} \cdot t + \frac{1}{2} \cdot a_y \cdot t^2 \quad v_{fy}^2 = v_{oy}^2 + 2 \cdot a_y \cdot y \quad y = \frac{v_{oy} + v_{fy}}{2} \cdot t$$

12. A ball is projected horizontally from the top of a 92.0-meter high cliff with an initial speed of 19 m/s. Determine: (a) the horizontal displacement, and (b) the final speed the instant prior to hitting the ground.

13. Determine the launch speed of a horizontally-launched projectile which lands 26.3 meters from the base of a 19.3-meter high cliff.

14. A soccer ball is kicked horizontally at 15.8 m/s off the top of a field house and lands 33.9 meters from the base of the field house. Determine the height of the field house.

15. A ball is projected at an angle with an initial horizontal velocity of 8 m/s and an initial vertical velocity of 29.4 m/s. The trajectory diagram shows the position of the ball after each consecutive second. Express your understanding of projectiles by filling in the blanks.



16. Determine ... (a) ... the displacement of the ball, (b) ... the height above the ground at its peak and (c) ... the final speed of the ball upon hitting the ground.

17. Suppose that the horizontal component of the initial velocity had been 13 m/s and the vertical velocity had been unchanged (in questions #15 and #16). Determine the ... (a) ... time of flight (b) ... the displacement of the ball, (c) ... the height above the ground, and (d) ... the speed upon hitting the ground.

Vectors and Projectiles

18. A physics student is driving his pick-up truck down Lake Avenue. The pick-up is equipped with a projectile launcher which imparts a vertical velocity to a water-filled rubber projectile. While traveling 20 m/s in an eastward direction, the projectile is launched vertically with a velocity of 60 m/s.

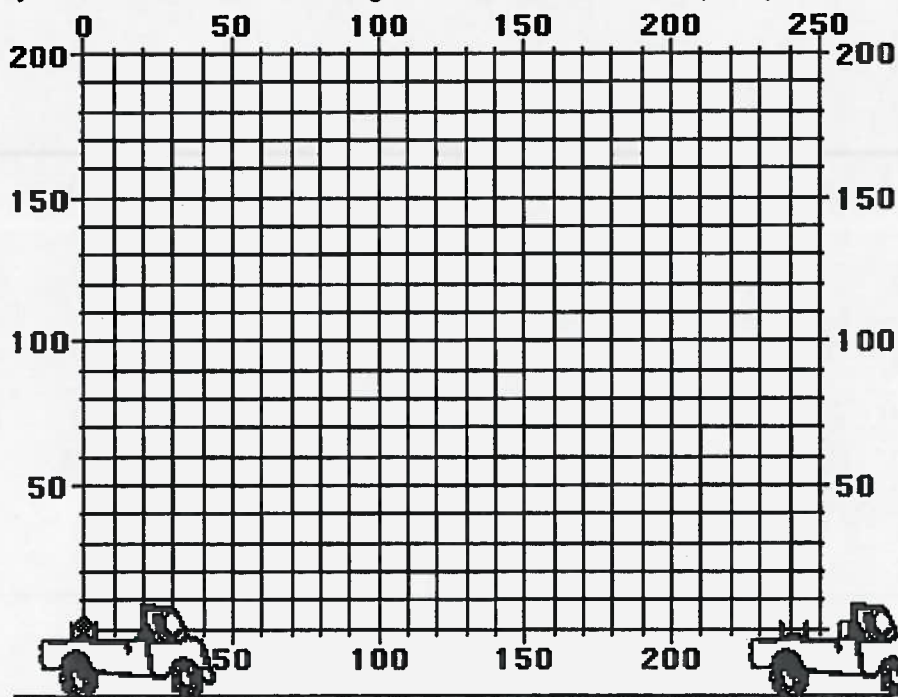
Fill in the table at the right, showing the horizontal and vertical displacement of the projectile every second for the first 12 seconds. Use the approximation that $g = 10 \text{ m/s}^2$, down.

$$x = v_{0x} t + \frac{1}{2} a_x t^2$$

$$y = v_{0y} t + \frac{1}{2} a_y t^2$$

| t (s) | x (m) | y (m) |
|-------|-------|-------|
| 0 | | |
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | | |
| 11 | | |
| 12 | | |

19. On the diagram below, place a large dot on the location of the projectile during each second of trajectory. Draw a smooth curve through the dots to indicate the trajectory.

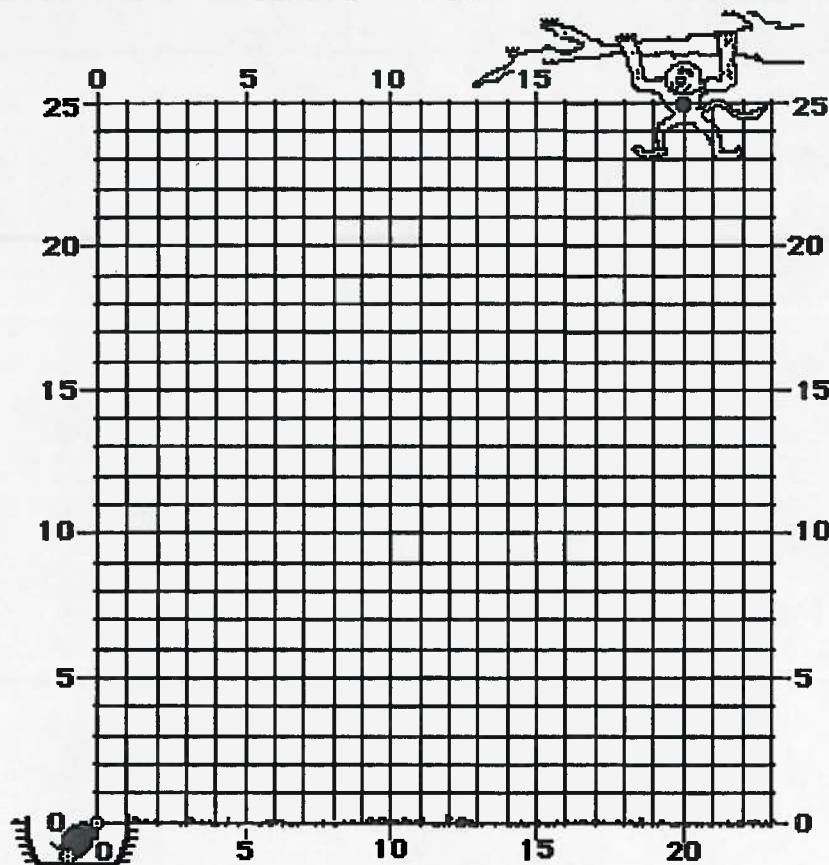


20. Will the projectile land in the truck, behind the truck or in front of the truck? (Assume no air resistance.) _____ Explain your answer.

21. A zookeeper has a monkey which he must feed daily. The monkey spends most of the day in the trees just hanging from a branch. When the zookeeper launches a banana to the monkey, the monkey has the peculiar habit of dropping from the trees the moment that the banana is launched. The banana is launched with a speed of 16.0 m/s at a direction of 51.3° above the horizontal (which would be directly at the monkey). The monkey is initially at rest in a tree 25.0-m above the ground. Use kinematic equations to determine the horizontal and vertical displacements of the banana and the monkey at 0.5-second time intervals. Then plot the trajectories of both banana and monkey on the diagram below. (Use the approximation that $g = 10 \text{ m/s}^2$, down.)

| Banana | | | |
|----------|--------|--------|------------|
| Time (s) | dx (m) | dy (m) | Height (m) |
| 0 | 0 | 0 | 0 |
| 0.5 | | | |
| 1.0 | | | |
| 1.5 | | | |
| 2.0 | | | |

| Monkey | | |
|----------|--------|------------|
| Time (s) | dy (m) | Height (m) |
| 0 | 0 | 25.0 |
| 0.5 | | |
| 1.0 | | |
| 1.5 | | |
| 2.0 | | |



22. Based on your mathematical analysis above, will the zookeeper hit the monkey if she aims the banana directly at the monkey? _____

Vectors and Projectiles

23. Use trigonometric functions to resolve the following velocity vectors into horizontal and vertical components. Then utilize kinematic equations to calculate the other motion parameters. Be careful with the equations; be guided by the principle that "perpendicular components of motion are independent of each other." PSYW

| | | |
|---|---|--|
| A long jumper leaps with an initial velocity of 9.5 m/s at an angle of 40° to the horizontal. | Megan Progress, GBS golf standout, hits a nine-iron with a velocity of 25 m/s at an angle of 60° to the horizontal. | A place kicker launches a kickoff at an angle of 30° to the horizontal and a velocity of 30 m/s. |
| $v_{ox} = \underline{\hspace{2cm}} \text{ m/s}$ | $v_{ox} = \underline{\hspace{2cm}} \text{ m/s}$ | $v_{ox} = \underline{\hspace{2cm}}$ |
| $v_{oy} = \underline{\hspace{2cm}} \text{ m/s}$ | $v_{oy} = \underline{\hspace{2cm}} \text{ m/s}$ | $v_{oy} = \underline{\hspace{2cm}}$ |
| $t_{up} = \underline{\hspace{2cm}} \text{ s}$ | $t_{up} = \underline{\hspace{2cm}} \text{ s}$ | $t_{up} = \underline{\hspace{2cm}}$ |
| $t_{total} = \underline{\hspace{2cm}} \text{ s}$ | $t_{total} = \underline{\hspace{2cm}} \text{ s}$ | $t_{total} = \underline{\hspace{2cm}}$ |
| $x = \underline{\hspace{2cm}} \text{ m}$ | $x = \underline{\hspace{2cm}} \text{ m}$ | $x = \underline{\hspace{2cm}}$ |
| $y @ \text{ peak} = \underline{\hspace{2cm}} \text{ m}$ | $y @ \text{ peak} = \underline{\hspace{2cm}} \text{ m}$ | $y @ \text{ peak} = \underline{\hspace{2cm}}$ |
| Show some work here: | Show some work here: | Show some work here: |

24. Generalize the calculations performed in question #23 above by writing the equations used to calculate each of the quantities requested in the problem.

$$\begin{array}{ll}
 v_{ox} = \underline{\hspace{2cm}} & v_{oy} = \underline{\hspace{2cm}} \\
 t_{up} = \underline{\hspace{2cm}} & t_{total} = \underline{\hspace{2cm}} \\
 x = \underline{\hspace{2cm}} & y @ \text{ peak} = \underline{\hspace{2cm}}
 \end{array}$$

25. Determine the range of a ball launched with a speed of 40 m/s at angles of (a) 40 degrees, (b) 45 degrees, and (c) 50 degrees from ground level. PSYW and label your answers.

26. For the three initial launch angles in question #25, determine the peak heights. PSYW and label your answers.